

Construction Methods and Equipment Management
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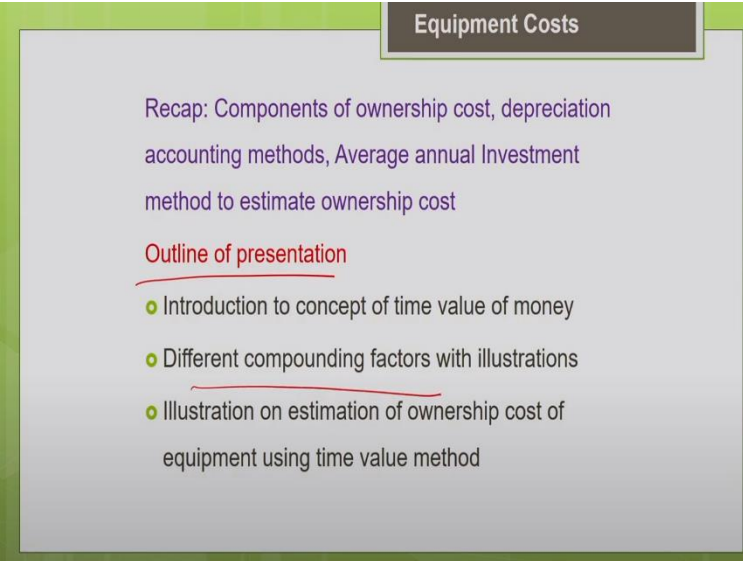
Module No # 01

Lecture No # 03

Equipment Cost – Ownership cost (Time Value Method)

Hello everyone. I welcome you all to the lecture 3 of this course construction methods and equipment management. So in this lecture we will be discussing about the equipment cost estimation specifically I will discuss about the ownership cost using time value method.

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Equipment Costs

Recap: Components of ownership cost, depreciation accounting methods, Average annual Investment method to estimate ownership cost

Outline of presentation

- Introduction to concept of time value of money
- Different compounding factors with illustrations
- Illustration on estimation of ownership cost of equipment using time value method

So let us have a recap of what we learnt in the previous lecture. So in the previous lecture I have introduced to you what are all the components of the ownership cost. We have discussed about the different depreciation accounting methods. Then we worked out a problem on how to estimate the ownership cost using average annual investment method. So this is the outline of today's presentation. In today's presentation I will just introduce to you what is the time value of money?

So as I told you earlier like the cash flows are occurring at different time period.. So we have to convert those cash flows with the occurring at the different time period into equivalent value at a particular time period using various compounding factors. So how to use those compounding factors we are going to discuss in this lecture with some illustrations. And we will also work out of an example on how to estimate the ownership cost using time value method in this lecture.

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Ownership Cost

Time value method

Concept of equivalence

- Time value of money is the difference between money borrowed for use today and returned at future date.
- If Rs 100 is borrowed at 8% interest
- After 1 year borrower will owe the original
- $100 + (100 \times \frac{8}{100}) = \text{Rs } 108$
- After 2 years borrower will have to owe
- $108 + (108 \times \frac{8}{100}) = \text{Rs } 116.64$
- Three amounts Rs 100 (today), Rs 108 (1 year) and Rs 116.64 (2 years) are equivalent.
- Economic equivalence is combination of interest rate and time value of money to determine different amounts of money at different time periods which are equal in economic value.
- Equivalent cash flows produce same result over specific period of time.

Rs 100, Rs 108, Rs 116.64
8% C.I.

Let us see what is this time value of money? As everyone knows that value money changes with time, it is known concept for everyone. So basically if you borrow say 100 rupees from the bank today. So if I am going to return that 100 rupees after 1 year I cannot return the same 100 rupees I have to add some interest to the particular principle 100 rupees. And I have to return back the principle plus interest together at the end of one year.

So how much will be the interest rate that will be decided by the bank depends upon the compounding rate of interest that followed by the bank policy. So according to the bank policy it will vary. So basically the bank or any borrower is charging you the interest for the money whatever he is lending you. The reason why they charge the interest everyone knows it is because they are taking risk by lending you the money.

For this particular risk they are charging the interest so that they can get the profit. So the interest nothing but the earning power of the money. So in another perspective you can say by giving the 100 rupees to the person, the person who is lending is losing the freedom of using, that 100 rupees. To compromise it he is charging you this interest kind of rent. It is kind of rent what we pay for using his money.

So in different perspective you can define it in different way. But basically the interest is the earning power of the money. So basically time value of money is the difference between the money

borrowed for use today and return at the future date. So when I borrow 100 rupees from the bank so if it is going to be compounded at the interest rate 8% then after 1 year I am supposed to return $100 + 8$ by 100 into the principle amount. This is your interest.

So the total money which you have to return after 1 year will be rupees 108. Say if I want able to return the money after 1 year I am going to return it after 2 years then what happens? Your interest will be compounded. Now you have to calculated in interest for the new principle 108. So $108 + 8$ by 100 into 108 so this is the amount which I have to return to the bank after 2 years. So it comes around 116.64. So the thing we have to keep in mind is all these three values are equivalent.

The 100 rupees today and rupees 108 after 1 year and rupees 116.64 after 2 years, all these are equivalent. They are not equal but economically they are equivalent. So this is called as economical equivalence. That is they are going to create the same effect only. So this is the concept of equivalence. They are made equivalent at the compounding interest say 8%. At 8 percent compounding interest they are made equivalent.

So, economic equivalent is the combination of the interest rate and; the time value of the money to determine different amounts of money at different time periods which are equal in the economic value. So basically, these equivalent cash flows will result in same result over a period of time. So the result is going to be same that is what we say here economically equivalent. Though they are not equal they are economically equivalent from economic perspective.

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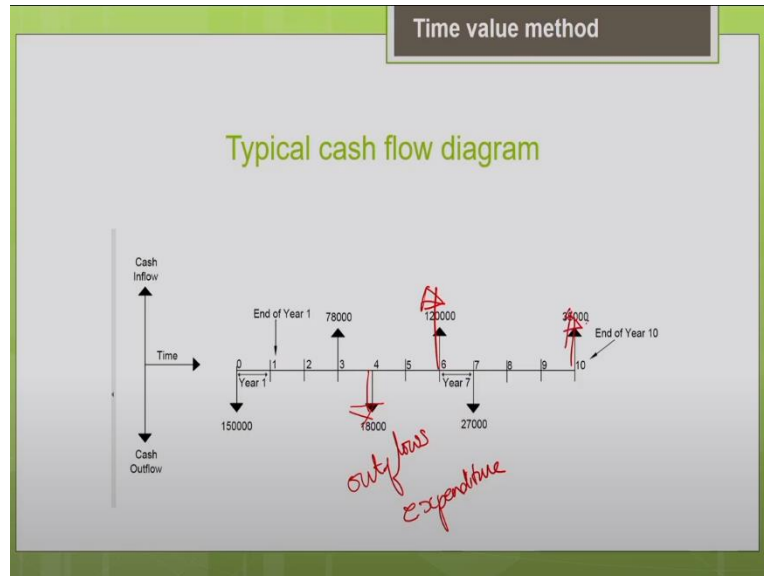
Time value method

- Understanding time value concept and compounding factors is a prerequisite to estimate ownership cost considering timing of cash flows
- Using time value concept we will learn how to convert cash flow occurring at one particular time period to equivalent value at another time period.
- Hence we can have rational comparison of cash flows at different time periods

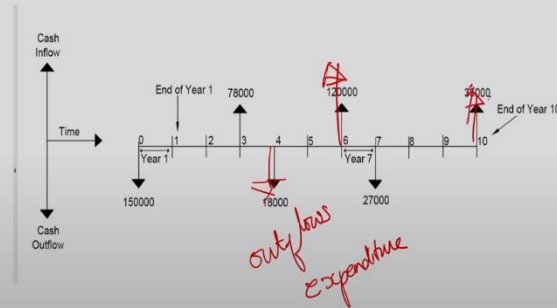
So in this time value method so basically you have to understand the time value concept and the compounding factors. So what are all the compounding factors which are generally adopted to convert the cash flows which are occurring at different time period to a particular time period to equivalent value or a particular period those compounding factors you should know. So this is a pre-request to estimate the ownership cost using time value method.

So that is it why we are going to learn what are the different compounding factors available and what are the application of this compounding factors and how to make use of them that is all we are going to discuss on the upcoming slides. Using time value concept, we will learn how to convert the cash flow occurring at one particular time period to equivalent value at another time period. So that is what the main purpose of this time value concept. So that we; can have a rational comparison of cash flows at different time periods which will be more sensible.

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Typical cash flow diagram

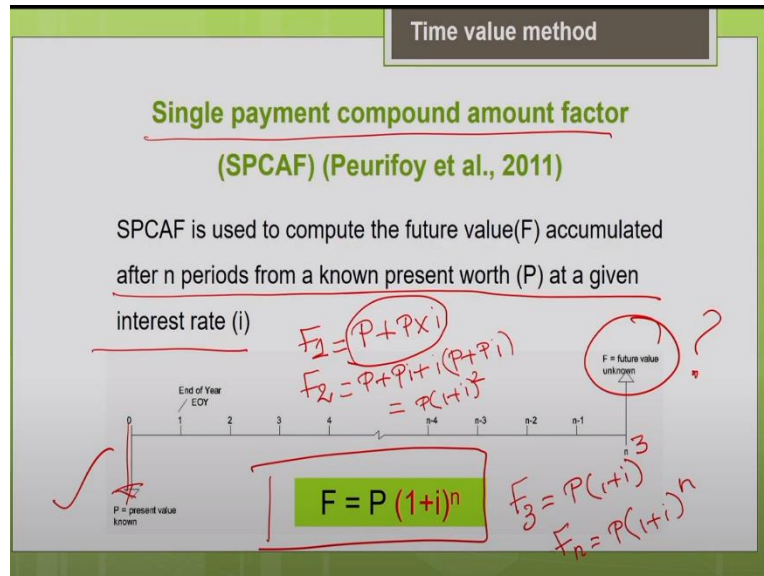


So this shows the typical cash flow diagram. Cash flow diagram is nothing but a graphical representation of the cash flows which are occurring at different point of time. So basically the horizontal line indicates the time period. The time 0, whatever the cash flow occurring at time 0 it indicates the present value. This is the present value year 1 2 3 4 everything indicated the end of the year. So you have remember, that year 1 2 3 means it indicates the end of the period.

And the cash flows which are pointing downward so they are cash outflows. So basically your expenditure is your cash out flow. So cash out flows expresses the arrow pointing downward. And cash inflows are represented as arrow pointing upward. So all your revenue, income your research so whatever is the cash inflow. Say for example if you are going to sell your equipment at the end of the useful life of the machine, the salvage value is your cash inflow.

So that will be represented as the arrow pointing upward. So these things you have to remember in mind. So we are going to use the cash flow diagram to discuss the compounding factors.

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So now let us move on to the first important compounding factor which we need to learn that is single payment compound amount factor. It is given as: -

$$F = P (1 + i)^n$$

Where,

F = Future value

P = Present value

I = Interest rate

n = period

So this single payment compound amount factor is used to compute the future value accumulated after n periods from a known present worth at a given interest rate i. That means if you know the present value of an asset we can find its future value.

So future value is unknown, you know the present value; you would like to estimate the future value of the particular asset. So you know that that the compounding rate of interest i and you know the n what is the life what is the period for which you are calculating. Then you can use this single payment compounding act factor to estimate the future value. So basically say for example if I am investing 1 lakh in bank.

So it is going to be compounded at the interest rate of say 8% for 8 years. So after 8 years what will be future sum of this present 1 lakh that it can calculate using this single payment compound amount factors. Say how will, you calculate basically the future value of the end of the first year

is nothing but you know the principle value P , now you have to calculate the interest for the principle. P into i , i is the compounding rate of interest.

Now for the end of the second year you need to find the future value. How will you calculate now? $P + P$ into i + you have to calculate interest for this entire new sum now. So i into $P + Pi$ so if you simplify it will be P into $1 + i$ whole square. Similarly, if you want to calculate the future value at the end of the third year F_3 you will get P into $1 + i$ the whole cube. So if you want to calculate the future value end of n th year it is P into $1 + i$ power n .

So this is how we get this formula. F is nothing but P into $1 + i$ power n . If I know the present value of a sum I can know its future value, at the end of n years, if n is known and the compound rate of interest i is known. So now from the equipment perspective we will see one example.

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Time value method

Problem

A twin engine scraper costs ₹82,00,000 this year. If the rate of inflation is expected to be 5% per year for the next several years, what will be the estimated cost to replace the machine at the end of 9 years?

A twin engine scraper machine it is costing 82 lakh this year, if the rate of inflation is expecting to be 5% per year for the next to several year. I hope everyone knows what is inflation? So inflation refers to the loss in buying power of the currency. So the rate of inflation is 5% what will be the estimate cost to replace a machine at the end of 9 years. So I know the present value of this equipment, I need to know the future value of the equipment at the end of 9 year.

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Time value method

Solution

$$F = P (1+i)^n$$
$$F = 82,00,000 (1+0.05)^9$$
$$= ₹1,27,20,891.37$$

So I can use the same the single payment compounding amount factor.

$$F = P (1 + i)^n$$
$$F = 82,00,000 (1+0.05)^9$$
$$= ₹1,27,20,891.37$$

I need to know the future value of the scraper, I know the present value of the scraper, I know the rate of inflation instead of interest rate I am taking rate of inflation here the life of the machine is 9 years. So it helps you to calculate the future value of the machine. So this compounding factor is very useful because you can know say after 9 years, if I want to replace this old machine with a new machine, how much amount of money I may need.

So that accordingly I can save that money in the pool of funds reserve for the equipment purchase. So this way it will be helpful in the equipment economics.

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Time value method

Present worth compound amount factor
(PWCAF)

If future amount F is given, present value P can be calculated by transposing the equation as below. P is present worth of F at interest rate 'i'

$F = P(1+i)^n$
 $P = \frac{F}{(1+i)^n}$

$P = \frac{F}{(1+i)^n}$

PWCAF = $\frac{1}{(1+i)^n}$

Now the next important factor which we are going to discuss is present worth compound amount factor. It is just an inverse of what we discuss so far. So what is the single payment compound amount factor? $F = P$ into $1 + i$ power n . So now to find the present value you just transpose equation. After transposing it becomes:-

$$P = \frac{F}{(1+i)^n}$$

This gives the present value. So in this case the present value is unknown but the future value is known. It is just an inverse of the earlier case.

Say for example if I want 10 lakhs after 8 years the compounding interest rate of 8%. How much they should deposit now in the bank. How much amount they should deposit now in the bank? So that it will grow to 10 lakhs at the end of 8 years at the rate of interest of 8%. So I can calculate using this formula. So to calculate the present value, I need the future value, the future value is known, n is known, i is known I can find the present value.

So you should know that the P is made equivalent to F by this factor.

$$PWAC = \frac{1}{(1+i)^n}$$

This factor is called present worth compounding amount factor.

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A twin engine Scraper is going to cost ₹1,27,20,891 after 9 years. How much money we need to keep aside today, so that money will grow to ₹1,27,20,891 in 9 years at compounding interest rate of 5% and enable you to buy the equipment?

$$\begin{aligned}
 P &= \frac{F}{(1+i)^n} \\
 &= \frac{1,27,20,891}{(1+0.05)^9} \\
 &= ₹ 81,99,999.76
 \end{aligned}$$

So let us workout another example from equipment perspective. A twin engine scarper is going to cost the same numbers I have used again, so that it will be more easier to understand. It is going to cost 1,27,20,891 rupees after 9 years. So you know its future value after 9 years. How much money we need to keep aside today, so that money will grow to 1,27,20,891 rupees in 9 years at a compounding interest of 5% and so that you can buy this particular equipment. So you know the future value of equipment after 9 years.

So how much money I should now deposit in the bank at the compounding interest rate of 5 % for 9 years. So that at the end of the ninth year I will, be able to buy this particular equipment. How to plan for that? So how much money I have to deposit now so that after 9 years I can buy this new equipment. So it is we are going to find out the present value. It is nothing but F divided by 1 + i power n. Future value is known compounding interest 5% number of years is 9, you can calculate.

$$\begin{aligned}
 P &= \frac{F}{(1+i)^n} \\
 P &= \frac{1,27,20,891}{(1+0.05)^9} = ₹ 81,99,999.76
 \end{aligned}$$

So it means now I have to deposit a amount of 81,99,999.76 rupees in the bank for 9 years at the interest rate of 5%. So that it will grow to 1,27,20,891 rupees so that at the end of ninth year I will be able to buy this particular machine.

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Time value method

**Uniform series compound amount factor
(USCAF)**

USCAF is used to determine the future sum (F) of a known annual series with uniform amount (A)

$$F = A \left[\frac{(1+i)^n - 1}{i} \right]$$

$$USCAF : \left[\frac{(1+i)^n - 1}{i} \right]$$

Let us see the next important compounding amount factor it is nothing but uniform series compound amount factor. So basically the payments most of the companies you can see it will be occurring cash flows, payments or the receipts all these are occurring in the series. So for example if you are buying an equipment through loan so doing loan repayment, so that will be in series. Similarly your receipts or revenue all these things will be receiving it in series the period of time.

So here we are going to consider only uniform series. Uniform series means in this series of cash flow all the cash flows are equal in value. So I noted very carefully it is equal. All the individual cash flows are equal value. A refers to the individual cash flow of this series of the cash flow and A is equal. So now what is a purpose of this uniform series compound amount factor? It helps us to determine the future sum of the known annual series with uniform amount A. It is represented as follows: -

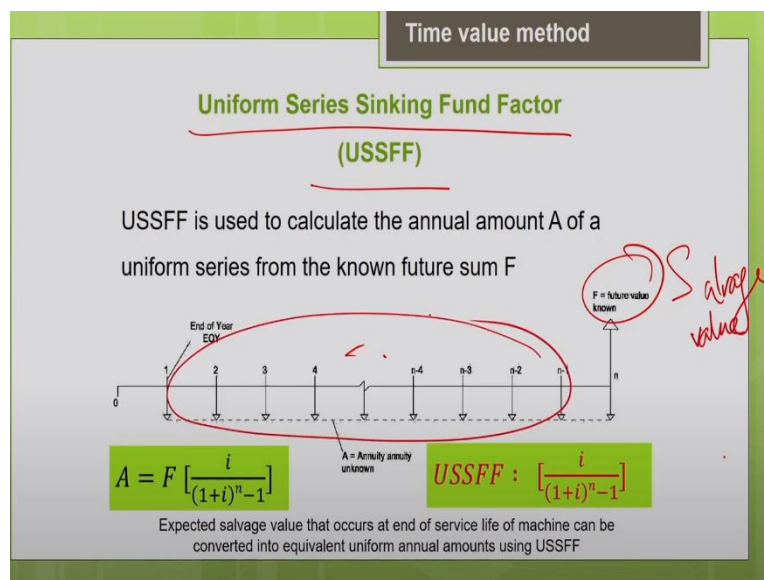
$$USCAF = \left[\frac{(1+i)^n - 1}{i} \right]$$

$$F = A \left[\frac{(1+i)^n - 1}{i} \right]$$

So you know the series. The series is known A is known. I need to find the future value of this A. F is unknown the future value is unknown. I need to find the future value of this A. Then I can make use of this uniform series compound amount factor. So basically say for example I am investing in bank let say at the end of every year I am investing 1 lakh in the bank for 8 years.

At the end of every year I am investing, I am depositing the same amount of 1 lakh, at the end of every year for 8 years at the rate of interest of say 8%. So at the end of 8 years how these series of cash flow grow into. So what is the future value this series of investments for the deposits for what you have made. That is what we are calculating here. So the series of investments or the deposits are known.

What is the future value of the series is unknown. So that is what they are calculated? So F is made equivalent to A by this factor. This is the factor called as uniform series compound amount factor. **(Refer Slide Time: 17:02)**



So another important factor is uniform series sinking fund factor. So it is an inverse of the earlier case what we have discussed. It is represented as: -

$$USSFF = \left[\frac{i}{(1+i)^n - 1} \right]$$

$$A = F \left[\frac{i}{(1+i)^n - 1} \right]$$

In this the series of the cash flow that is unknown. The uniform series A is unknown but your F is known. So this is used to calculate the annual amount A of the uniform series from the known future sum F. So future sum F is known, I am going to find the equivalent A for it.

So it is just obtaining the transpose of the earlier equation. This equation if you transpose it you will get the A value. So you are going to find the A from F, i and n. All these are known this is unknown. So basically how much amount of money I should deposit in bank. Every year for 8 years same example 8 years, how much amount of money I should deposit for 8 years. Every year I should deposit same amount at the compounding interest rate of 8%.

So that at the end of the eighth year say I will be getting 10 lakhs this is known. So I need 10 lakhs at the end of eighth year. So every year how much I should deposit in the bank at the compounding interest of 8%. This A is unknown that is what we are calculating here. So from equipment percent if we see it helps you to calculate what is the equivalent value of the salvage value?

So you know that salvage value is one which is occurring at the end of the useful life of the machine it is a future value. So at the end of the useful life of the machine you are disposing the machine or selling it at the reasonable price that is what is your salvage value of the machine? Ok. So how to convert this salvage value which is occurring at the future into equivalent cash flows into equivalent uniform annual amounts.

So we can do that using this factor uniform series sinking fund factor. So this is one application in the equipment economics point of view. So we are converting the salvage value which is occurring in future into equivalent uniform annual amounts using uniform series sinking fund factor.

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Time value method

Problem

How much must be deposited at the end of each year into a sinking fund earning 10% compounded annually in order to accumulate ₹1,27,20,892 for equipment replacement at the end of 9 years?

USSFF ✓
F A ? ✓

Now let us see an illustration from the equipment perspective. So much must be deposited at the end of each year into a sinking fund earning 10% compounded annually in order to accumulate 1,27,20,892 rupees for equipment replacement at the end of 9 years. So, basically say when we plan for the purchase of a equipment or replacement of old machine with a new machine. So generally we do not prefer to use the entire revenue of one complete year one particular year for the equipment purchase.

Generally most of the companies will not follow the particular policy because we cannot spend the entire revenue of the company of one year for the equipment purchase itself. There will be lot of other commitments so it may not be possible for us to do that way. And equipment cost may be very significant. So generally what they do is they plan for equipment purchase or equipment replacement in a systematic way.

So there will be particular pool of funds dedicated for the equipment purchase or equipment replacement, for this sinking fund. In the particular pool of fund is they will deposit money. So every year they will be depositing some amount of money. Say for n years this will be compounded at the particular rate of interest say i percentage. So it will help you to either buy a new machine or replace old machine with new machine at the end of the particular period.

So that is how we plan for this equipment purchase or equipment replacement. We deposit some amount of money in pool of funds called as sinking fund that is what is this problem about? How much must be deposited at the end of each year into a sinking fund earning. So how much we deposit into sinking fund earning. The rate of interest is 10% so that my future value is needed is 1,27,20,892.

So that at the end of 9 years I will, be able to replace my old machine with the new machine. So this I can go for uniform series sinking fund factor, with the help of this factor I can estimate this value. So you know the future value is known. You need to know what is the A , every year what should I deposit into the sinking fund. Using this compounding factor, I can estimate that.

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Time value method

Solution

$$A = F \left[\frac{i}{(1+i)^n - 1} \right]$$

$$A = 1,27,20,892 \left[\frac{0.1}{(1+0.1)^9 - 1} \right]$$

$$= ₹9,36,773.34$$

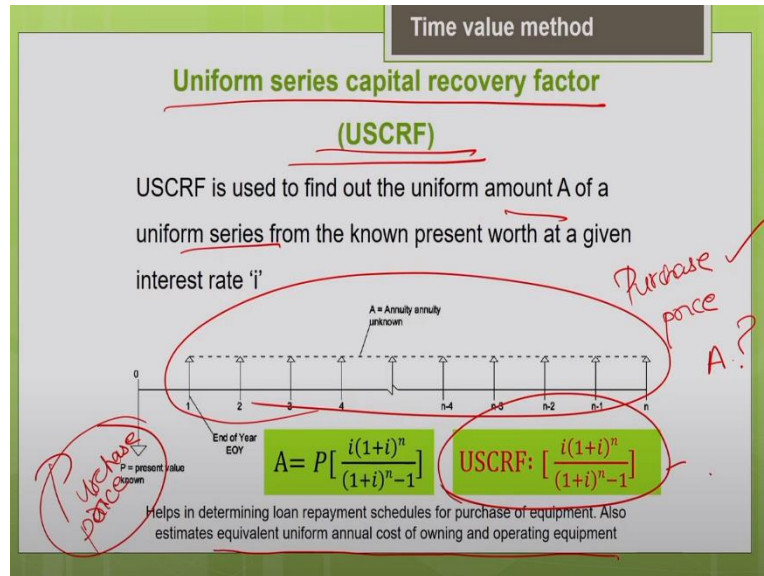
This is your uniform series sinking fund factor.

$$A = F \left[\frac{i}{(1+i)^n - 1} \right]$$

$$A = 1,27,20,892 \left[\frac{0.1}{(1+0.1)^9 - 1} \right] = ₹9,36,773.34$$

A is unknown, F is known, i is known, n is known using this we calculate. Now you know that every year I should deposit 9,36,773.34 rupees into the sinking fund. So that at the end of 9 years I will be able to generate an amount of 1,27,22,892 with that I will be able to replace my old machine with a new machine.

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So the next important factor which we are going to discuss today is uniform series capital recovery factor. It is given as follows: -

$$USCRF = \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

$$A = P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

Basically this is used to find out the uniform amount A of a known series from the known present worth at a given interest rate i . So you know the present worth of a particular asset. The present value is known. How to convert it into a equivalent uniform series. P the present value is known. The present value is known. How to convert it into equivalent uniform series? That is possible using this uniform series capital recovery factor.

Say for example if I deposit 10 lakhs in bank now. So for a period of 9 years I have deposited at a interest rate of say 9%. For 9 years I have deposited at the interest rate of 9%. So what will be amount of money which I will get at the end of every year as a return. So that return is going to be in the form of equal amount of money. So that is what is A ? That I can find it if I know that I am going to deposit 10 lakhs now at the end of every year how much amount of money I will get from the bank as a return in terms of equivalent amount.